

Modelling – Deterministic vs. Stochastic

September 19, 2025

Introduction

Many actuaries and investment consultants provide trustees with simple projections to show where the financial status of the plan is heading. However, in many cases, these projections are based on simple assumptions which everyone knows will be wrong, but they are used for decision making purposes. The problem is these simple projections tend to hide risks from trustees and may lead the trustees into making uninformed decisions with dire consequences.

In this paper we will focus on how different types of models may lead to very different decision-making outcomes.

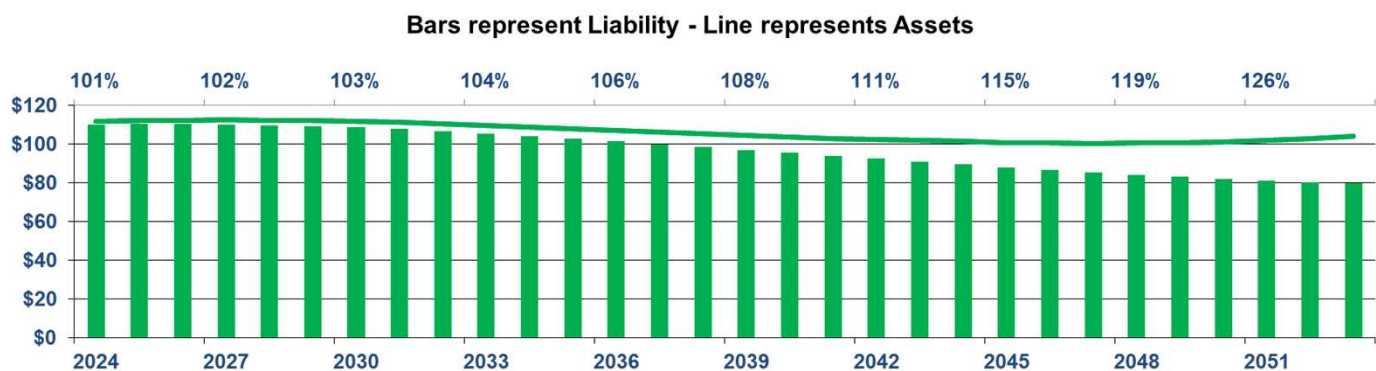
The Illusion of Static Returns: Stress Testing Pension Sustainability

The bar chart below presents financial projections for a pension plan from 2024 to 2051, focusing on the relationship between assets and liabilities:

- Green bars represent liabilities, which gradually decrease over time.
- A green line tracks assets, which remain relatively stable with a slight upward trend toward the end of the projection.
- Percentages above the chart (ranging from 101% to 126%) indicate the funded ratio, the proportion of assets to liabilities, suggesting the plan remains well-funded throughout the period.

Modeling the Mirage: Why 7% Every Year Is Unrealistic

This projection assumes a static return environment of 7.0% annually, which explains the smooth asset growth and improving funded status. However, this scenario is unrealistic in practice due to market volatility and sequence of returns, risk, especially for plans with significant negative cash flow. In this plan, the Hudson Ratio is \$8 paid out for every \$1.50 contributed.



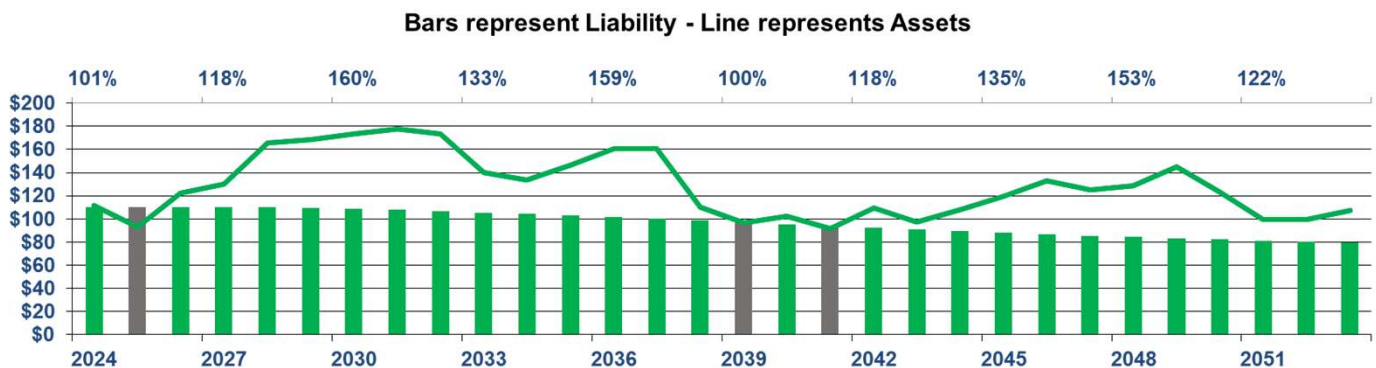
Question: Which scenario is better, the one with a lower average return (5.75%) or the one with a higher average return (8.26%)?

Scenario 1: 5.75% Average Return

- Includes early positive returns followed by volatility and some negative years.
- The plan experiences temporary underfunding but ultimately ends up overfunded.
- Demonstrates resilience due to favorable early returns.

PYB	Return	PYB	Return
2024	-10.93%	2039	14.79%
2025	40.97%	2040	-2.96%
2026	13.18%	2041	28.32%
2027	34.60%	2042	-4.37%
2028	6.44%	2043	19.10%
2029	7.84%	2044	17.78%
2030	7.00%	2045	17.96%
2031	2.54%	2046	-0.40%
2032	-14.68%	2047	8.20%
2033	1.32%	2048	18.22%
2034	15.94%	2049	-11.00%
2035	15.89%	2050	-14.55%
2036	5.00%	2051	6.30%
2037	-27.25%	2052	14.16%
2038	-4.92%	2053	-0.96%
		AVE	5.75%

Scenario 1: Asset vs. Liability Ratios by Year



As you can see, there are a couple of years where the plan becomes underfunded, but long-term, the plan ends up being overfunded.

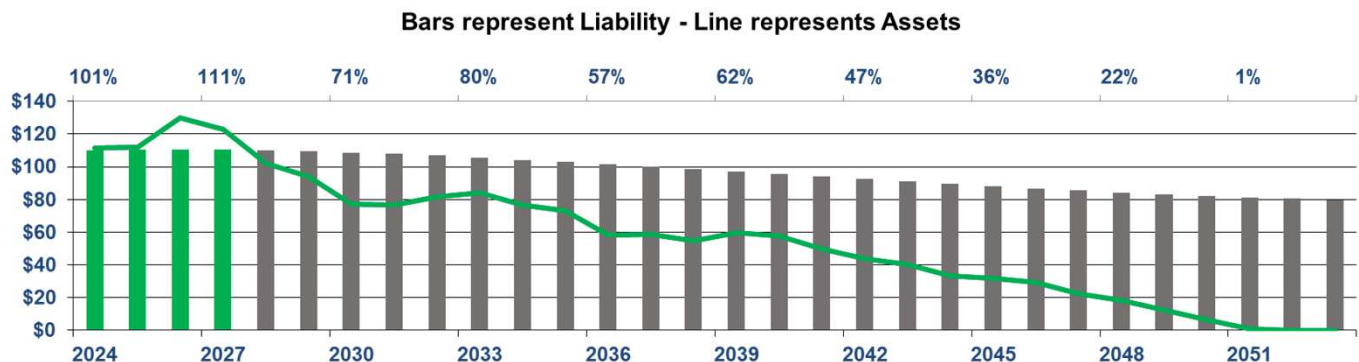
Now let's look at Scenario 2, which had a higher average return during the projection period.

Scenario 2: 8.26% Average Return

- Despite a higher average, if early returns are poor, the plan will be forced to sell assets at a loss.
- This will lock in losses, reduce the asset base, and impair long-term sustainability - even if later returns are strong.

PYB	Return	PYB	Return
2024	6.65%	2039	10.56%
2025	23.63%	2040	-0.73%
2026	0.37%	2041	3.72%
2027	-11.01%	2042	10.49%
2028	0.03%	2043	0.24%
2029	-10.38%	2044	19.83%
2030	10.14%	2045	16.18%
2031	18.35%	2046	0.63%
2032	14.01%	2047	15.13%
2033	1.22%	2048	3.28%
2034	6.51%	2049	3.45%
2035	-10.35%	2050	17.80%
2036	16.28%	2051	22.18%
2037	7.71%	2052	9.20%
2038	25.27%	2053	33.14%
		AVE	8.26%

This series of returns resulted in the following projections.



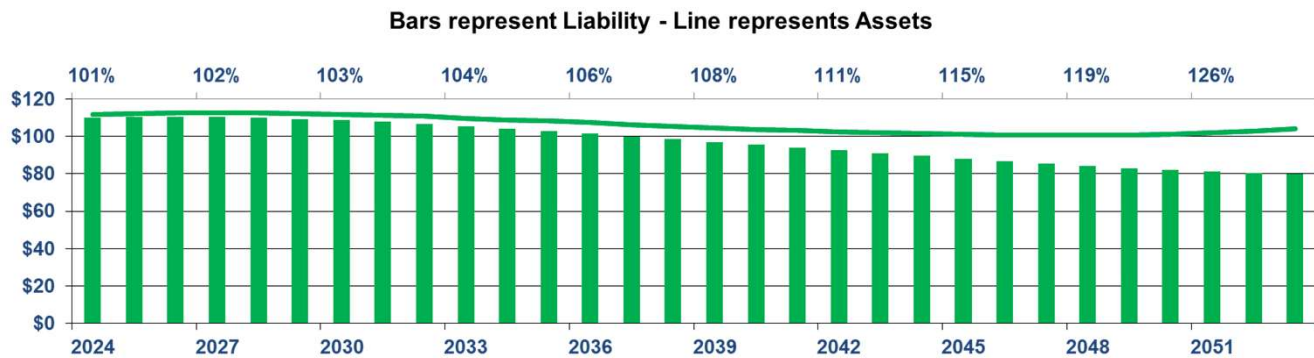
Conclusion: As you can see, higher returns are not always better – for a mature plan, the timing of when you get the returns is more important. **Scenario 2** seems better due to its higher average return. But in pension plans with **negative cash flow**, the **timing of returns** matters more than the average. **Sequence matters more than average.** A lower average return with favorable early performance can outperform a higher average return with early losses, especially for plans that are cash flow negative and must liquidate assets to pay benefits.

Contribution Risk

One possible reason for ignoring this risk is that a pension plan can get additional contributions to make up the funding shortfall if needed. Let's take a look at how contributions could be impacted. Going back to our original chart, we start with the following:

- Currently **fully funded**
- Contributing **12.5% of pay** (about \$1.5 million annually)
- Paying out **\$8 million** in benefits and expenses annually

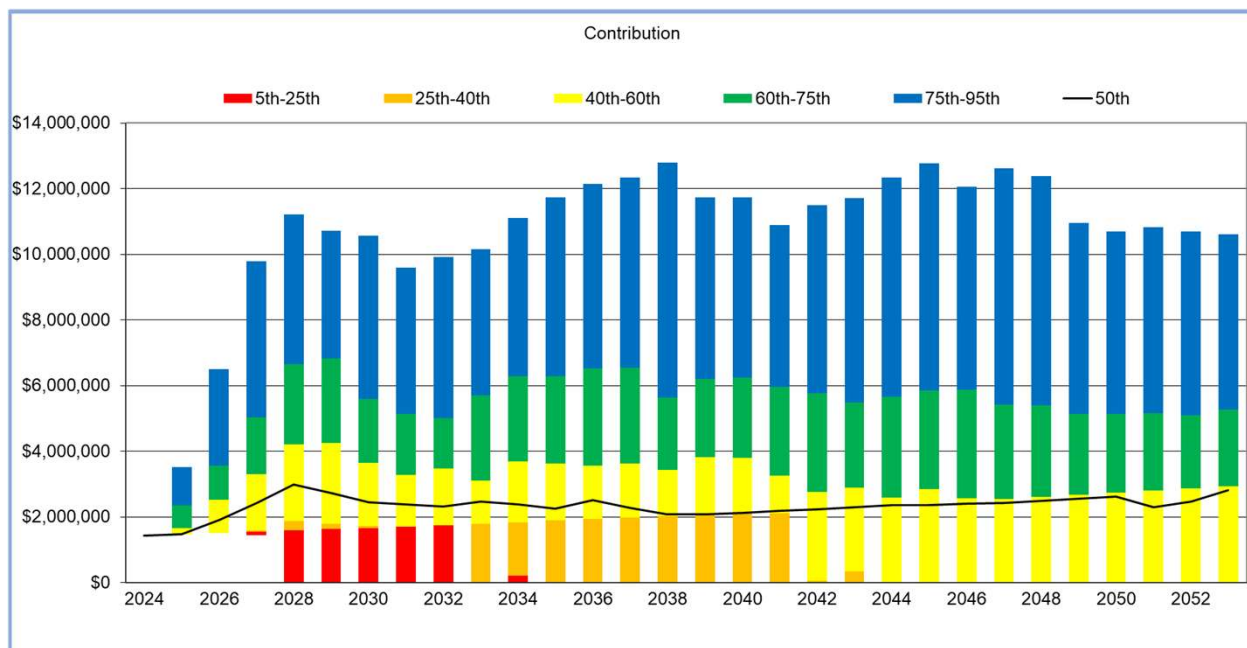
So, is this plan well-funded, and are the contributions that are being made sufficient for this plan?



Unfortunately, these very simple projections give plans a false sense of security. Given that many plans are in a negative cash flow position, the impact of volatile markets on the plan is hidden.

As stated earlier, this particular plan has about \$8 million going out in benefits and expenses and about \$1.5 million coming in with contributions.

If we run a stochastic forecast, we get the following expected contribution numbers for the plan.



The Contribution chart presents a stochastic forecast of pension plan contribution levels from 2024 to 2052. Here's a breakdown of its structure and meaning:

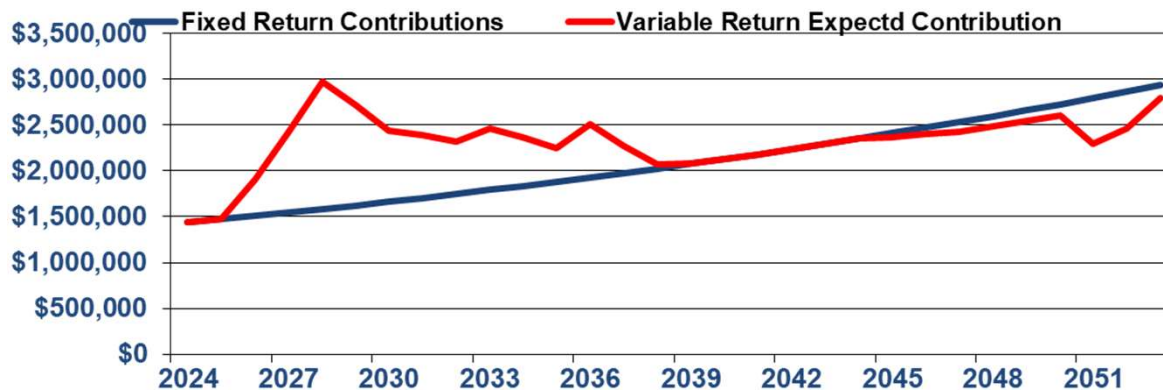
Chart Overview

- X-Axis: Years from 2024 to 2052
- Y-Axis: Contribution amounts (in dollars)
- Colored Bars: Represent contribution percentiles for each year:
 - Red: 5th–25th percentile
 - Orange: 25th–40th percentile
 - Yellow: 40th–60th percentile
 - Green: 60th–75th percentile
 - Blue: 75th–95th percentile
- Black Line: Represents the 50th percentile (expected contribution level)

Higher Contributions. The chart shows a wide range of possible contribution outcomes each year, with a noticeable spike in expected contributions (black line) occurring in the near future. This spike highlights the impact of market volatility and negative cash flow, which may require significantly higher contributions than projected under fixed-return assumptions.

Now we will graph the expected contributions from the stochastic projections against the contributions made, assuming a static 7.00% annual return.

Projected Contribution Paths: Fixed vs. Variable Return Scenarios



The above line graph compares two pension contribution scenarios over time from **2024 to 2051**:

- Y-Axis: Contribution amounts in dollars
- X-Axis: Years from 2024 to 2051

Blue Line – Fixed Return Contributions:

- Represents contributions under a fixed 7.0% annual return assumption.
- Shows a steady and gradual increase over time due to salary inflation

Red Line – Variable Return Expected Contribution:

- Represents contributions under a stochastic forecast with variable market returns.
- Shows significant fluctuations, with periods of sharp increases and dips.
- Overall, it trends upward, indicating higher contribution needs due to market volatility and negative cash flow.

The data clearly illustrates a substantial and prolonged increase in contribution levels under realistic return variability. This raises a critical question: Can the sponsor sustain a doubling, or more, of contributions in the near future? If not, the plan may face severe funding stress, potentially leading to insolvency or collapse. This is typical of any plan that has significant negative cash flow, where the impact of market volatility is often overlooked.

Conclusion

The **Fixed vs. Variable Return Scenarios** chart underscores the inherent danger in relying on fixed return assumptions when evaluating pension funding strategies. While the fixed return model may suggest a stable and manageable contribution path, the variable return scenario exposes the true volatility and potential for sharp increases in required contributions. These spikes, particularly under adverse market conditions, can place significant financial strain on plan sponsors and stakeholders.



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